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TECHNICAL TRANSLATION

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EFFECT OF SLIGHT BLUNTING OF LEADING EDGE OF
AN IMMERSED BODY ON THE FLOW AROUND IT
AT HYPERSONIC SPEEDS

By G. G. Chernyi

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EFFECT OF SLIGHT BLUNTING OF LEADING EDGE OF AN IMMERSED BODY ON THE FLOW AROUND IT AT HYPERSONIC SPEEDS*

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In the present paper, an attempt is made to extend the theory of the flow around slender bodies with tapered leading edges at hypersonic speeds [1, 2] to cases where the leading edge of the immersed body is slightly blunted. This generalization of the theory is of great import, since it is impossible to achieve ideally sharp leading edges of slender airfoils or ideally sharp leading edges of airframes, in reality. Even with the most painstaking fabrication of models, the leading edges will be several microns thick and, after the models have been immersed for a short period in a supersonic flow, the thin leading edges of the models will become degraded and acquire a thickness of the order of 20 microns. In the case of large-scale objects, it is hardly possible to speak of the thickness of the leading edges as being less than one or several tenths of a millimeter.

But it is not only due to difficulties in manufacturing technology and in the strength of the materials that ideally sharp leading edges on airfoils and airframes are impossible to achieve in practice. At hypersonic flight speeds, the thin leading edges would inevitably melt away because of the impossibility of bleeding off the large quantities of heat liberated in the flow of gas adjacent to the leading edge of the body through the thin tips.

Accordingly, instead of bodies with ideally tapered leading edges, we have to deal in reality with bodies that are slightly blunted at the leading edge. It is just such blunted bodies, in which the dimensions of the blunted portion are small compared to the longitudinal dimensions, that we propose to discuss in the present contribution.

In the case of hypersonic flow around a blunt-nosed body, a detached shock wave forms ahead of the body, with a subsonic zone adjacent to the shock. This circumstance renders the theoretical study of such flow

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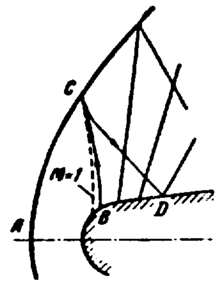
patterns extremely difficult, especially when it is considered that the small characteristic size of the blunted portion may make it necessary to take into account gas viscosity effects in the vicinity of the leading edge of the body.†

Fig. 1.



One attempt has been made to elaborate a semi-empirical method for taking the effects of leading-edge bluntness in slender airfoils into account, in flow at moderate supersonic speeds [5]. The basic concept underlying this approach may be illustrated with the aid of Fig. 1. The flow in the region between segment AC of the shock and the extreme characteristic BC, extending from the surface of the body and having a common point with the acoustic-speed line, will be the same for different airfoils having a different shape of tip bluntness. Neglecting the interaction of disturbances proceeding along the characteristics in the direction toward the body with disturbances proceeding outward from the immersed body, the pressure on the surface of the body to the right of the point B may be obtained by superposing the pressure computed for the airfoil on the basis of Busemann's formula on the pressure associated with the disturbances approaching the body, which are independent of the airfoil geometry, at least over the segment BD, where those disturbances are most intense. Accordingly, in order to compute the pressure distribution over different airfoils having the same tip bluntness geometry, it is sufficient to have at hand data on the flow pattern over one such airfoil, e.g. a plate. Experimental data on the pattern of flow over a plate having a leading edge of elliptic shape with an aspect ratio ranging from 0 (rectangular leading edge) to 8, and for the range of Mach numbers from 1.4 to 1.8, are to be found in [6]. An analysis of the results shows that consideration of the effects of slight tip blunting

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† Available experimental data [3, 4] point to a significant dependence of the flow pattern near the blunted leading edge of a plate on the Reynolds number at Re values, computed with the aid of the characteristic size of the blunt tip of the order of several thousands.

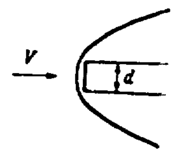
on the flow over an airfoil at moderate supersonic speeds introduces only slight corrections into the results of the theory of flow over airfoils with sharply tapered leading edges. The drag on a blunt-nosed airfoil may be obtained in straightforward fashion by adding the blunt-tip drag (obtained, say, experimentally) and the drag on the remainder of the airfoil, computed in accord with theory of flow around tapered bodies, without taking into account drag effects contributed by disturbances proceeding from the subsonic region, since these are small in intensity.

However, the limited extent of the region of complex flow near the
F blunted leading edge of the body, compared with the characteristic dimen-
3 sions of the body, does not always serve as a basis for neglecting effects on
5 the flow pattern on the scale of the entire body. Experimental data [4] and theoretical considerations (see below) attest to that fact that, at hypersonic speeds of flow, a slight blunting of the leading edge of a plate may significantly alter the flow pattern and pressure distribution of the flow in a region whose dimensions exceed by hundreds and thousands of times the dimensions of the blunted portion per se.

In that case, we can count on obtaining a good approximation to the description of the phenomena taking place in flow around slightly blunt-tipped bodies, if we neglect the distortion of the body due to blunting, but allow for the effect on the flow, replacing it by the effect of the concentrated forces applied to the flow by the blunting. The problem of flow over slender blunt-nosed bodies at hypersonic speeds was formulated in that manner by the author in a previous paper [7] and was developed further in a number of other contributions [8, 9].

1. Statement of the problem. Observing the statement of the problem as indicated above, we treat the motion, in a gas at a velocity $\underline{V}$, of a body in which all forward-facing surface elements form small angles with the direction of flow. We may consider as an exception the case where a small leading portion of the body is blunted. The size of the blunted portion will be assumed so small that it may be neglected when treating the flow in a region having dimensions of the order of the longitudinal extent of the body. The effect exerted by the blunted tip on the flow, manifesting itself over a large region despite the small size of the blunted portion, is here replaced by the effect of concentrated forces applied to the gas on the part of the blunted tip. The magnitude of the concentrated forces may be assumed known from experimental data or from a theoretical treatment of the flow pattern in the vicinity of the leading edge of the body. At hypersonic speeds, these forces may be determined to an approximation, e.g. by Newton's formula.

We restrict ourselves to cases of symmetrical flow around airfoils or around bodies of revolution. In the first case, we treat the pattern of flow in the upper half plane (in the layer between two closely spaced parallel planes), and in the second case we treat the flow pattern in the meridional plane above the axis of symmetry (in the angular region between two closely spaced planes passing through the axis of the body).

Fig. 2. 

The effects of the blunted tip on the gas in the layer so delineated is replaced by the resultant of those forces applied to the gas on the part of the blunted tip in the direction of flight, and the resultant of those forces applied to the gas in the direction perpendicular to the direction of flight. We may designate as X and Y, respectively, those resultants referred, in planar flow, to a layer of unit width, and in axis-symmetrical flow, to the layer at an angle 2π to the axis of symmetry. In computing the total forces replacing the effect of the blunted tip of the flow, the excess pressure forces must be taken into account, and in some cases the forces of viscous friction, since gas viscosity may exert an appreciable effect on the flow pattern in the neighborhood of the leading edge (in the case of a blunted tip of very small size, the action of viscosity may be of the same order as the effect of pressure forces, or significantly in excess of them). In flow around a blunt-tipped airfoil with a detached shock wave, the magnitude of forces X and Y must be extended to include the excess pressure forces (and the forces of viscous friction) acting on the gas from the direction of that portion of the plane of symmetry located between the departing shock wave and the leading edge of the immersed body.

The force X, acting in the direction of flow of the body, does work on the gas, imparting energy to it. The energy of the gas in the layer of unit width normal to the direction of flight increases as a result of the action of the blunted tip by an amount $E=X \cdot 1$. The resultant force Y does no work but, like the force X, imparts momentum to the gas. The momentum imparted to the gas by the blunted tip in the direction perpendicular to the direction of flight, in the same layer of unit width, is equal to $I=Y/V$.

We now make use of the equivalence, established in [1, 2, 10], between the problem of hypersonic gas flow around slender bodies and the problem of nonsteady laminar gas flow (the law of planar cross sections). For a blunt-nosed slender body, the equivalent problem of nonsteady flow consists in the following:

In a gas initially at rest, an energy E is liberated at a certain instant of time at the plane (on a straight line), and a momentum I is transmitted to the gas at a normal to that plane (straight line); the energy E and the momentum I are referred, respectively, to unit area and unit length of charge. At the same instant of time, a flat-headed (round cylindrical) piston begins to expand at a rate of travel U in the gas from the point of energy liberation. The motion taking place is to be determined. For the transition from the problem formulated for unsteady flow to the problem of steady flow over a body in the direction of the x-axis at a speed of flow V, we must put $E=X$, $I=Y/V$, $U=V \tan \alpha$ (where α is the angle of inclination of an element of the profile contour or of the body of revolution to the $\underline{x}$ direction), and the time $\underline{t}$ is introduced by means of the relation $x=Vt$.

An exact analytic solution of the problem, for $E \neq 0$, has been obtained [11] only for the case where the effect of the initial gas pressure on the flow may be safely neglected, and $I=0$, $U=0$ (problem of a powerful explosion). The flow then taking place is progressive in character and corresponds to the hypersonic flow pattern over a blunt-headed plate (in the case of laminar flow) or over a round cylinder with the flow perpendicular to the end-face surface (in the case of axis-symmetric flow). Under more general conditions, an exact solution of the problem may be found in each concrete case only by use of complicated numerical techniques, similar to those used
F to find the solution of the problem of a point blast [12-14]. The problem
3 may be solved approximately, for example, with the aid of the method,
5 described in [15], of expanding the solution into a series in powers of $(\gamma-1)/(\gamma+1)$, γ being the specific-heat ratio.

2. Flow around a plate with a blunt leading edge and around a round cylinder with its end face normal to the oncoming flow. Consider (see Fig. 2) the pattern of flow of a gas at hypersonic speeds over a flat plate of thickness d having a blunted leading edge (we may also consider an infinitesimally thin plate, but with a finite viscous friction force on the small segment next to the leading edge). In that case, in the equivalent problem of one-dimensional unsteady flow with plane waves, we must assume $E \neq 0$, $U=0$, i.e., the problem of motion developing in a gas at rest in response to the explosion of a charge distributed over the plane must be treated. The parameters defining such a flow are: the initial gas pressure p_0, the initial density ρ_0 the blast energy E (referred to unit charge area), the specific-heat ratio γ of the gas, the distance r from the plane of the explosion, and the time t. Since only three independent nondimensional combinations of those parameters may be set up, e.g.,

$$\gamma, \quad p_0 r/E \quad \text{and} \quad p_0^{3/2} t/\rho_0^{1/2} E,$$

the fundamental theorem of the theory of similitude and dimensionality [11] stipulates that all of the variables to be determined, after reduction to nondimensional form, will be functions of only those three parameters. Replacing t and E according to the formulas

$$t = x/V \quad \text{and} \quad 2E = 2X = c_x \, 1/2 \, \rho_0 \, V^2 d$$

(where c_x is the drag coefficient on the blunt tip), we find that in hypersonic flow over a flat blunt-headed plate the dimensionless quantities to be determined are functions solely of the variables γ, $x/(c_x M^3 d)$, and $r/(c_x M^2 d)$. Thus, for example, for the pressure distribution over the surface of the plate, i.e. at $r=0$, the formula

$$\frac{\Delta p}{p_0} = P\left(\frac{1}{c_x M^3}\,\frac{x}{d}\,\gamma\right) \tag{2.1}$$

is valid.

This formula indicates, in particular, that the extent of the region subject to increased pressure in the neighborhood of the leading edge of the plate increases very sharply as the Mach number increases (proportionally to M^3). The shape of the leading surface of pressure discontinuity arising in response to flow over the blunt-headed plate is determined by the relation

$$\frac{1}{c_x M^2}\,\frac{r^*}{d} = R\left(\frac{1}{c_x M^3}\,\frac{x}{d}\,\gamma\right) \tag{2.2}$$

Functions P and R may be found by numerical solution of the problem of explosion, requiring the use of high-speed computers; however, as indicated above, up to the present time the solution has been obtained only for the case of flow patterns exhibiting spherical waves (blast waves originating at a point) for $\gamma = 1.4$. For a high-intensity shock, when the initial gas pressure p_0 is negligibly small compared to the pressure downstream of the shock, the pressure cannot affect the flow pattern. Accordingly, in that case the parameter p_0 as well as the Mach number are immaterial, so that Eq. (2.1) and (2.2) must assume the form

$$\left.\begin{aligned} \frac{\Delta p}{\frac{1}{2}\rho_0 V^2} &= \kappa(\gamma)\, c_x^{2/3}\left(\frac{x}{d}\right)^{-2/3} \\ \frac{r^*}{d} &= \kappa_1(\gamma)\, c_x^{1/3}\left(\frac{x}{d}\right)^{2/3} \end{aligned}\right\} \tag{2.3}$$

Functions $\kappa(\gamma)$ and $\kappa_1(\gamma)$ cannot be determined solely from considerations derived from the theory of similitude and dimensionality; their values

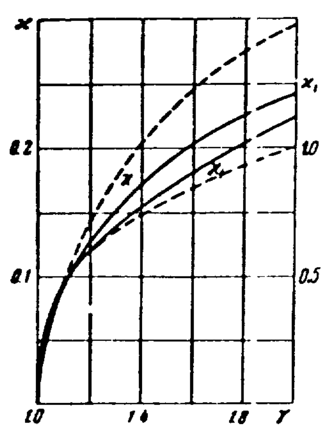


Fig. 3.

may be obtained from an exact solution of the problem of the explosion of a flat charge, with the initial pressure data left out of account [11]. These values represented by the solid lines in Fig. 3.

The right-hand members of Eqs. (2.3) constitute the principal terms in the quantities $\Delta p/(1/2\ \rho_0 V^2)$ and r^*/d, for small values of the variable $x/(c_x M^3 d)$.

The following terms in that notation are to be found in [16, 17]. In the paper referred to above [15], we find an approximate solution to the problem of the explosion of a flat charge by the method of expanding the solution into a series in powers of $(\gamma-1)/(\gamma+1)$.

Compare the results of the theory outlined above, expressed by Eqs. (2.1) and (2.2), with the more exact calculations of the flow over a flat plate with a blunt leading edge [18] and with the available empirical data. Figure 4a shows values obtained by the method of characteristics, at Mach numbers of 5.00, 6.86, and 9.50, for the pressure on a flat plate whose leading edge presents the form of a wedge with an angle at the vertex such that the speed of flow aft of the attached shock wave forming is exactly equal to the speed of sound. The graph also gives the pressures on a plate with a hemi-cylindrical leading edge, at Mach 14, also computed by using the method of characteristics [19]. Values of $x/(c_x M^3 d)$ and $\Delta p/p_0$ are laid out along the horizontal and vertical axes, in accord with

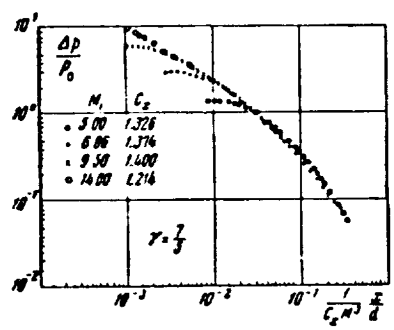


Fig. 4a.

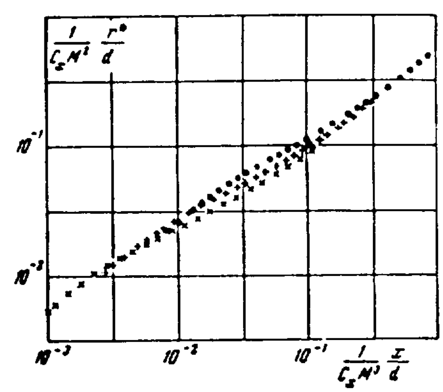


Fig. 4b.

Eq. (2.1) (for a wedge-shaped tapered leading edge, c_x is readily found from the equations for oblique shocks, and for the half-round edge, c_x is taken from the refined Newton equation [20] as equal to $2/3\ c_x{}^*$). Eliminating the small area around the break point of the immersed contour, all of the pressure distributions plotted in those coordinates coincide. Fig. 4b. The shape of the shock waves corresponding to the cases cited of flow over a plate with a wedge-shaped tapered leading edge. Here again, starting with a slight distance from the leading edge, all the curves are in good agreement.

The results cited demonstrate that the law of planar cross sections may be used in studying flow over slender blunt-nosed bodies at hypersonic

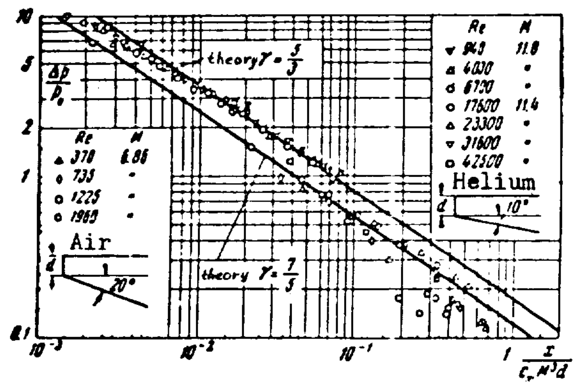


Fig. 5.

speeds. The pressure data obtained from Eq. (2.3) agree satisfactorily with the major portion of the curves computed by the method of characteristics, up to values of $x/(c_x M^3 d)$ roughly equal to 1/100.

Figure 5 illustrates the results of two series of experiments on flow at hypersonic speeds past blunt-edged plates. The first series of experiments was performed in a helium-filled wind tunnel at Mach numbers of the order of 12 [4]. The model used was a 10° wedge, one side of the wedge was made in the form of a flat cross section normal to that side of the wedge. The second series of experiments was carried out in a wind tunnel operating on air at about Mach 7, with a similar model, but with an apex angle of 20° [21].

In the first series of experiments, experimental values of $\Delta p/p_0$ and of $r^*/(c_x M^2 d)$ as a function of $x/(c_x M^3 d)$ for different plate thicknesses and different Mach data on oncoming flow showed good agreement and closely confirmed the theoretical equations (2.3) at $\gamma=5/3$ when the Reynolds number, arrived at on the basis of the plate thickness, exceeded 5000-6000. When the Reynolds number was reduced below that range of values, the effect of viscosity on the flow pattern in the vicinity of the leading edge took on importance, and the value of c_x that had to be adopted in working up the experiments to secure agreement had to be sharply increased.

During the second series of experiments, the Reynolds number did not exceed 2000, and its effect was felt over the entire range investigated. At the highest Reynolds number, the experimental pressure data showed excellent agreement with Eq. (2.3) at $\gamma=7/5$, assuming $c_x=c_p^*$. At smaller Reynolds numbers, c_x rose sharply, as in the first series of experiments.

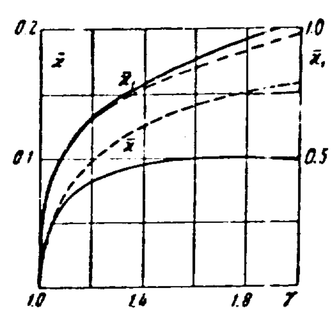


Fig. 6.

Thus the experimental findings likewise confirm the conclusions based on the theory outlined; it turns out that the effect of viscosity can be neglected in determining the value of c_x on the blunted edge, if the Reynolds number exceeds 2000-6000 (the data available does not permit us to establish the Re number with greater precision).

Note further that, according to the theory (Fig. 3), the effect of the blunted tip falls off as the ratio of specific heats γ is reduced; this fall-off is not very pronounced as γ varies from 5/3 to 7/5. The results of the experiments (Fig. 5) confirm this conclusion derived from the theory.

We may now proceed to a consideration of longitudinal flow at some hypersonic speed V past a round cylinder of diameter d having a blunted forward part. Repeating the same reasoning employed for flow over a flat plate, we find that in this case the flow is determined by the dimensionless parameters γ, $x/(\sqrt{c_x}M^2d)$, and $r/(\sqrt{c_x}Md)$. In particular, the pressure distribution over the surface of the cylinder and the shape of the shock wave are determined by the equations

$$\frac{\Delta p}{p_0} = P\left(\frac{1}{\sqrt{c_x}M^2}\,\frac{x}{d}\;\gamma\right) \qquad \frac{1}{\sqrt{c_x}M}\,\frac{r^*}{d} = R\left(\frac{1}{\sqrt{c_x}M^2}\,\frac{x}{d}\;\gamma\right) \tag{2.4}$$

At very high hypersonic speeds, these equations take on the form

$$\frac{\Delta p}{\frac{1}{2}\rho_0 V^2} = \bar{\kappa}(\gamma)\sqrt{c_x}\,\frac{d}{x} \qquad \frac{r^*}{d} = \bar{\kappa}_1(\gamma)\,c_x^{1/4}\left(\frac{x}{d}\right)^{1/2} \tag{2.5}$$

Graphs of the functions $\bar{\kappa}(\gamma)$ and $\bar{\kappa}_1(\gamma)$, plotted by using the exact solution for the problem of the powerful explosion of a linear charge [11] are shown in the solid lines in Fig. 6.

The equations derived point to the existence of a high-pressure area near the blunted leading end of the cylinder. At very high hypersonic speeds, the extent of the high-pressure area increases with the square of the Mach number (when the Reynolds number does not affect the magnitude of c_x); as γ decreases, the size of that area is also reduced.

Unfortunately, we do not have data on the pressure distribution over the surface of the cylinder and on the shape of the shock wave at distances many times in excess of the diameter of the cylinder and at high Mach numbers, for comparison with the results of the theory.

To conclude the present section, we may note that the solution of the problem of blast from a flat or linear charge also describes the pattern of flow around an arbitrary profile or body of revolution in a region whose dimensions are large compared to the transverse dimensions of the body. To describe the pattern of flow in a region where the shock wave has lost its strength, we must, of course, use the solution that takes the initial gas pressure into account.

3. Flow around a thin wedge with a blunted leading edge. As the simplest example of hypersonic flow around an airfoil with a blunt leading edge, consider the flow around a thin blunt-edged wedge. For that case, in the equivalent problem of unsteady gas flow with plane waves, $E \neq 0$, $U = V \tan \alpha = \text{const} \neq 0$ (where α is the half-angle of the wedge taper). This flow is not "progressive" even when initial gas pressure is small enough to be neglected, compared with the pressure aft of the shock. An approximate solution of the problem may be obtained with the aid of the method involving expansion of the solution into a series in powers of $(\gamma-1)/(\gamma+1)$ outlined in [15]. However, taking into account the fact, that, in the general case, even this method turns out to be rather laborious, let us simplify the method still further, enabling us to obtain a solution by elementary means that retains a satisfactory degree of accuracy. F35

The core idea of the method of expanding the solution into a series in powers of $(\gamma-1)/(\gamma+1)$ is that when the gas in the shock wave is strongly compressed, the bulk of the gas in the perturbed region is concentrated in a thin layer adjacent to the shock wave. The principal change in the gas pressure takes place precisely in that layer, whereas in the rest of the region (which may not even exist) the change in pressure due to the low gas density is extremely small. In order to obtain the solution in elementary form, we assume that the thickness of the layer next to the shock, which involves the entire mass of the gas, is negligible and that the change in pressure in the perturbed region outside that layer may be neglected as well.

Then, applying the energy equation to the gas within the perturbed region, we obtain:

$$\frac{1}{2}\rho_0 v\left(\frac{\partial R}{\partial t}\right)^2 + \frac{p}{\gamma - 1}(v - v_0) = E = \frac{p_0 v}{\gamma - 1} + \int_0^t p \, dv_0(t) \tag{3.1}$$

Here, $\underline{v}$ is the volume bounded by the surface of the shock wave, and v_0 is the volume swept out by the piston. In the case of plane waves, $\underline{v}$ is the distance from the plane of the blast to the shock wave, and v_0 is the distance to the piston. To use this equation to find the law of propagation of the shock wave $R_0(t)$, and with it all of the characteristics of the flow (according to [15]), we may use the principal terms in the expansions of functions of R and p in powers of $(\gamma-1)/(\gamma+1)$, i.e. we may assume

$$\frac{\partial R}{\partial t} = \dot{R}_0 \qquad p = \frac{2}{\gamma + 1}\rho_0 \dot{R}_0^2 + \rho_0 \frac{R_0 \ddot{R}_0}{\nu}$$

(the dot denotes differentiation with respect to time, $\nu = 1$ for plane waves, and $\nu = 2$ for cylindrical waves). However, in order to confer an elementary character on the whole theory, we use the momentum equation as a second equation for determining the functions $R_0(t)$ and $p(t)$. This equation has the following form:

$$\rho_0 v \frac{\partial R}{\partial t} = I + \int_0^t \left(p - p_0\right) S \; dt \qquad (3.2)$$

where S is the surface area of the shock wave; in the case of plane waves, $S=1$. In Eqs. (3.1) and (3.2), the value of $\partial R/\partial t$ represents the velocity of the gas, and is the same for all gas particles. We assume that the velocity of flow of the gas particles over the entire layer is the same as at points adjacent to the shock, i.e. we shall suppose that:

F 3 5

$$\frac{\partial R}{\partial t} = \frac{2}{\gamma + 1}\left(\dot{R}_0 - \frac{a_0^2}{\dot{R}_0}\right) \qquad (3.3)$$

(despite the poorer accuracy, we may assume $\partial R/\partial t = \dot{R}_0$; note that all of the formulations presented above agree with each other when the gas in the wave is infinitely compressed, i.e. at $\gamma \to 1$, $p_0 \to 0$).

Equations (3.1) and (3.3) enable us to determine the functions $R_0(t)$ and $p(t)$ for a given law governing the travel of the piston, $v_0(t)$. In the case of flow over a wedge, $v_0 = Ut$, $v = R_0$, and $S=1$.

We restrict ourselves now, for the sake of simplicity, to the case where the effect of the initial pressure on the flow may be neglected. Eliminating the pressure p from Eqs. (3.1) and (3.2), we obtain a single equation for the law governing the propagation of the shock wave (the subscript 0 of function R is omitted).

$$\rho_0 R \frac{1}{2}\left(\frac{2}{\gamma + 1}\dot{R}\right)^2 + \frac{1}{\gamma - 1}(R - Ut)\frac{d}{dt}\left(\rho_0 R \frac{2}{\gamma + 1}\dot{R}\right) = E - JU + \rho_0 U R \frac{2}{\gamma + 1}\dot{R}$$

We shall defer to later the case $U=0$, considered in a more accurate formulation in Section 2, and now introduce the scale $L=(E-IU)/\rho_0 U^2$ as a measure of length, the scale L/U as a measure of time, and the scale $\rho_0 U^2$ as a measure of the pressures.

The equation written above now takes on the form:

$$\frac{1}{\gamma - 1}(R - t)\frac{d}{dt} R\dot{R} = \frac{\gamma + 1}{2} + R\dot{R} - \frac{1}{\gamma + 1} R\dot{R}^2 \qquad (3.4)$$

This equation has the unique solution $R^*(t)$, satisfying the condition $R(0)=0$ and existing for $t \geq 0$. At small values of t:

$$R^* = \left[\frac{9}{4}\frac{(\gamma + 1)^2(\gamma - 1)}{3\gamma - 1}\right]^{1/3} t^{2/3}$$

At large values of t, R^* tends to the asymptote:

$$R = \frac{\gamma + 1}{2} t + (\gamma - 1) \tag{3.5}$$

which is an exact solution of Eq. (3.4.).

Since $R^*\dot{R}^* = 0(t^{1/3})$ at $t \to 0$, the solution $R^*(t)$ corresponds to the case I=0. For small values of the half-angle α at the apex of the wedge, the value of IU=Y tan α is small compared to the value of E=X (Y is of the order of X or less). This enables us to use the solution R^* to evaluate the effect of the blunted tip of the thin wedge on the pattern of flow past it at hypersonic speeds. Proceeding to the variables characterizing steady flow, we find that the shape of the shock is governed by the equation:

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$$\frac{r^*}{d} = \frac{c_x}{4 \tan^2\alpha} R^*\left(\frac{4 \tan^3\alpha}{c_x} \frac{x}{d} \gamma\right) \tag{3.6}$$

We obtain the following equation for the pressure distribution over the wedge

$$\frac{p}{\rho_0 V^2 \tan^2\alpha} = \dot{W}\left(\frac{4 \tan^3\alpha}{c_x} \frac{x}{d} \gamma\right) \tag{3.7}$$

where $W(t,\gamma)$ is used to designate the function $R^*\dot{R}^*(\gamma+1)^{-1}$. Graphs of Eqs. (3.6) and (3.7) are shown in Fig. 7. The position of the shock in flow past a sharp-edged wedge is also illustrated in Fig. 7.

It follows from Eq. (2.5) that the direction of the shock wave produced in flow past a blunt-tipped wedge tends to the same direction as in flow past a sharp wedge as we proceed further downstream, though the shock is displaced farther from the surface of the wedge. This additional displacement is due to the appearance of a region of rarefied gas near the surface of the wedge, and is:

$$\frac{\gamma - 1}{4} \frac{c_x}{\tan^2\alpha} d$$

according to Eq. (3.5), i.e. it may be substantial for slender wedges.

The absence of experimental data prevents us from making a detailed comparison between the results of calculation and the experimental results. Fig. 8 (see p. 13)is a schlieren interfergram of the flow of helium ($\gamma=5/3$) past a blunt-tipped 10° wedge [4], at Mach 12.7 and Reynolds number 15,000.

Figure 9 shows, for purposes of qualitative comparison, the flow pattern obtained by calculations based on Eq. (3.6) for $M=\infty$ and $\gamma=7/5$.

Fig. 7.

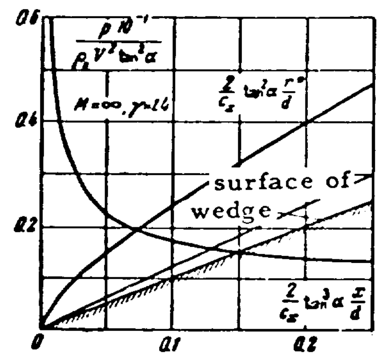
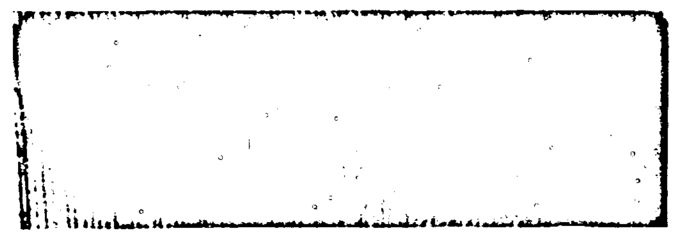


Fig. 8.



In the upper half plane, the oncoming flow is directed along the surface of the wedge, which corresponds to a value of $\alpha = 0$ in Eq. (3.6). Making use of the asymptote of the function R^* for small values of t, we find that, at $\alpha=0$:

$$\left.\begin{aligned} \frac{p}{\frac{1}{2}\rho_0 V^2} &= \left[\frac{\sqrt{2}}{3}\,\frac{(\gamma+1)^{1/2}(\gamma-1)}{3\gamma-1}\right]^{2/3} c_x{}^{2/3}\left(\frac{d}{x}\right)^{2/3} \\ \frac{r^*}{d} &= \left[\frac{9}{16}\,\frac{(\gamma+1)^2(\gamma-1)}{3\gamma-1}\right]^{1/3} c_x{}^{1/3}\left(\frac{x}{d}\right)^{2/3} \end{aligned}\right\} \qquad (3.8)$$

In order to judge the accuracy of these equations, the broken line in Fig. 3 is used to give the values of the first factors in the right-hand members of the equations, corresponding to the functions $\kappa(\gamma)$ and $\kappa_1(\gamma)$ in Eqs. (2.3), obtained in the exact solution of the probelm of gas flow past a blunt-edged plate at $M=\infty$ in the statement of the problem as presented.

We now calculate the total drag X on a wedge of length l with a blunted leading edge:

$$X^{\vee} = 2X + 2\int_0^l p \tan\alpha \, dx = 2X + 2XW\left(\frac{4\tan^3\alpha}{c_x}\,\frac{l}{d}\right)$$

The drag coefficient for that wedge is expressed by the equation:

$$c_x^{\vee} = \frac{2}{t}\left[1 + W(t)\right]\tan^2\alpha \qquad \left(t = \frac{4}{c_x}\tan^3\alpha\,\frac{l}{d}\right)$$

A graph for this relationship is plotted in Fig. 10 for small values of t. At large values of t, the approximate relationship:

$$c_x^{\vee} = \left(\gamma + 1 + \frac{2\gamma}{t}\right)\tan^2\alpha$$

As the equations obtained above demonstrate, a slight blunting of the leading edge results in a significant increase in the drag coefficient c_x in the case of thin wedges. Thus, the drag on a blunt wedge with a taper half- F
angle of 6° is twice the drag on a sharp wedge even at $l/d = 500$. Accordingly, 3
the leading edges must be tapered down with great care to avert a sharp in- 5
crease in drag on airfoils and stabilizing vanes in flow at hypersonic speeds. However, as indicated at the beginning of the article, it is hardly possible to meet this requirement in practice.

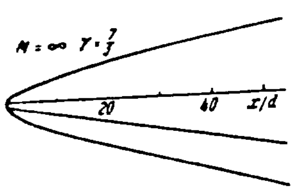


Fig. 9.

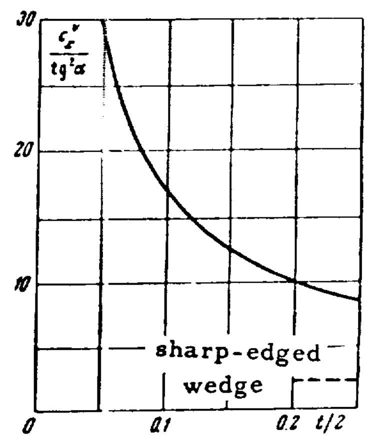


Fig. 10.

It should also be borne in mind that the center of pressure of an airfoil with a blunt leading edge is displaced forward, compared to the center of pressure of the same airfoil with a sharp edge. This displacement may be a sizeable one. Thus, for an airfoil in the form of a plate with a blunt leading edge, at high hypersonic speeds, Eq. (2.3) indicates that the center of pressure is situated at 1/4 of the length of the plate from the leading edge, and not at the midline of the chord, as in an infinitesimally thin plate.

In concluding the present section, let us point out that the approximate solution outlined for the case of flow past a thin sharp wedge yields

$$\beta = \frac{\gamma + 1}{2}\alpha \qquad \frac{p}{\rho_0 V^2} = \frac{\gamma + 1}{2}\alpha^2$$

(β=the angle of inclination of the shock to the direction of the oncoming flow), which agrees with the equations derived from the exact theory for $M=\infty$.

4. Flow over a thin blunt-nosed cone. Using the same approximate statement of the problem as in the preceding section, let us consider the problem of the flow pattern past a blunt-nosed cone. In that case, we must set $v=\pi R^2$, $v_0=V^2t^2$, and $S=2\pi R$, in the energy and momentum equations (3.1) and (3.2). We employ Eq. (3.3), to obtain the speed $\partial R/\partial t$ of the particles in the perturbed region, i.e. we allow for the initial pressure of the gas.

Assuming $U\neq 0$, we now introduce the scale $L=(E/\pi\rho_0 U^2)^{1/2}$ as a measure of length, and the scale L/U as a measure of time, and we define $p-p_0=\rho_0 U^2\Delta p$. Then Eqs. (3.1) and (3.2) assume the following form:

$$\left.\begin{aligned}
&\frac{1}{2}R^2\left(\frac{2}{\gamma+1}\right)^2\left(\dot{R}-\frac{1}{K^2\dot{R}}\right)^2+\frac{R^2-t^2}{\gamma-1}\left(\Delta p+\frac{1}{\gamma K^2}\right)\\
&\qquad\qquad = 1+\frac{R^2}{\gamma-1}\,\frac{1}{\gamma K^2}+2\int_0^t\left(\Delta p+\frac{1}{\gamma K^2}\right)t\,dt\\
&R^2\,\frac{2}{\gamma+1}\left(\dot{R}-\frac{1}{K^2\dot{R}}\right)=\frac{IU}{E}+2\int_0^t \Delta p\,R\,dt
\end{aligned}\right\}\qquad(5.1)$$

Here, $K=U/(\gamma p_0/\rho_0)^{1/2}=M\tan\alpha$, the parameter of similarity for hypersonic flow. At small values of t, the initial energy associated with the gas in the perturbed region and the work done on the gas by the piston are small compared to the energy liberated in the explosion, and the solution of the set of equations (5.1) becomes an approximate solution of the problem of a powerful blast that sets in motion cylindrical blast waves (the value of IU/E must then be assumed to be negligible, as in section 3:

$$\left.\begin{aligned}
R &= \left[\frac{4(\gamma+1)^2(\gamma-1)}{3\gamma-1}\right]^{1/4} t^{1/2}\\
\Delta p &= \sqrt{\frac{\gamma-1}{4(3\gamma-1)}}\; t^{-1}
\end{aligned}\right\}\qquad(5.2)$$

These equations readily yield Eq. (2.5), with the approximate values $\bar{\kappa}(\gamma)$ and $\bar{\kappa}_1(\gamma)$. These values are given by the broken lines in Fig. 6.

Equations (5.1), which are of the asymptotic form (5.2) for small values of t, may be solved by numerical integration. It follows from these equations that the functions $\dot{R}$ and Δp approach a constant value (corresponding to flow past a sharp-nosed cone) for large values of t:

$$\dot{R}\to\sqrt{\frac{\gamma+1}{2}+\frac{1}{K^2}}\qquad \Delta p\to 1$$

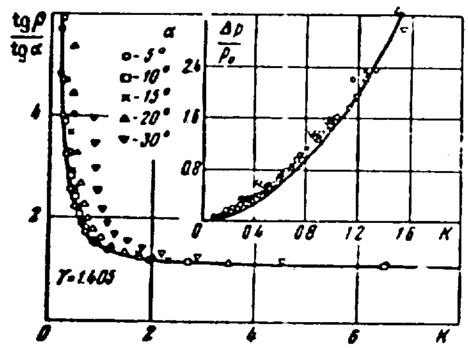


Fig. 11.

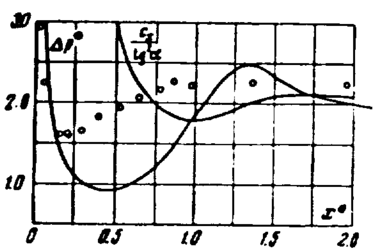


Fig. 12.

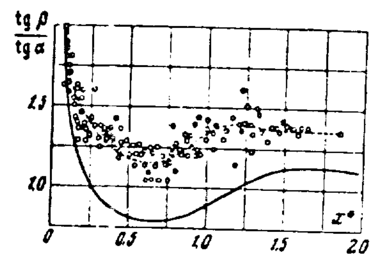


Fig. 13.

Fig. 11 shows graphs of the relationships derived from these equations:

$$\frac{\tan\beta}{\tan\alpha} = \sqrt{\frac{\gamma + 1}{2} + \frac{1}{K^2}} \qquad \frac{\Delta p}{p_0} = \gamma K^2$$

(β=the angle made by the chock with the direction of oncoming flow) and compares them with the exact values [22].

Calculations performed for the case where $K=\infty$ (i.e. neglecting the initial gas pressure) disclosed the following interesting features of the behavior of the solution. The pressure coefficient on the cone, which equals infinity at the leading point, decreases rapidly as we move along the generatrix of the cone, in some segment reaching values that are considerably below those recorded for a sharp-nosed cone having the same angle of taper (see curve in Fig. 12), where:

$$\Delta p^* = \frac{\Delta p}{\frac{1}{2}\rho_0 V^2 \tan^2\alpha} \qquad x^* = \sqrt{\frac{2}{c_x}}\,\frac{x}{l}\,\tan^2\alpha$$

Accordingly, the angle β between the shock and the direction of oncoming flow also has a minimum (curve in Fig. 13) This qualitative

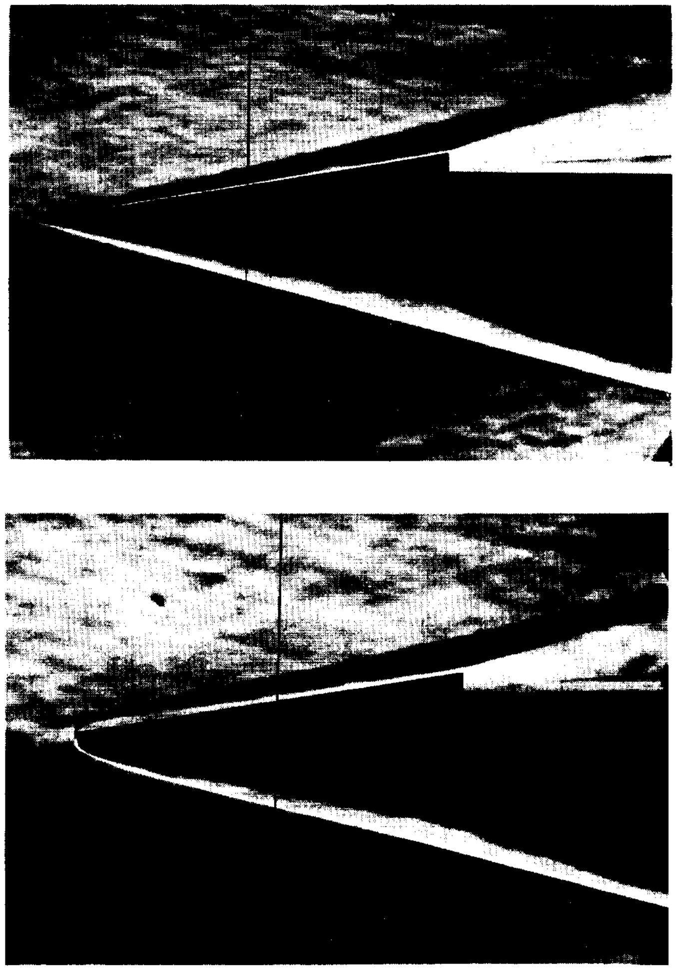


Fig. 14.

feature of the flow pattern is retained for values of the similitude parameter K of the order of unity, as is evidenced by experimental data secured [23] in flow over a blunt-nosed cone with a 10° half-angle of taper at Mach 6.85 (i.e. at $K=1.2$). Fig. 12 and Fig. 13 are plots of the results of these experiments, while Fig. 14 shows photographs of the flow past blunt- and sharp-nosed cones.

Since the pressure on an appreciable portion of the surface of a blunt-nosed cone is less than that on the surface of a sharp-nosed cone, the total drag on a blunt-nosed cone may well prove to be less than the drag on a sharply tapered cone.

The drag coefficient for a blunt-nosed cone (for $K=\infty$) is:

$$c_x^{*} = \frac{1}{2x^{*2}}\left[1 + 2\int_0^{2x^*} \Delta pt\ dt\right]\tan^2\alpha \qquad \left(\frac{l}{d} \approx \frac{0.96}{\tan^2\alpha}\sqrt{\frac{c_x}{2}}\right)$$

The drag coefficient for the blunt-nosed cone has a minimum at the value of l/d that is indicated within the parentheses; the relative decrease in drag compared to that on a sharp-nosed cone is as much as 10% (Fig 12).

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